NONSTEADY HEAT CONDUCTION IN CELLS OF TRIANGULAR LATTICE WITH CENTRAL HEAT SINK

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A method of numerical solution is outlined for the nonsteady heat conduction in the cells between the tubes of a triangular lattice with a central heat sink for nonlinear boundary conditions. Calculated and experimental data are compared.

In heat-transfer equipment in nonsteady conditions it is often required to calculate the heat conduction in the cells between the tubes. In the general case the appropriate model is a complex structure consisting of heat carrier in an intertube space with pores formed by hexagonally packed tubes (channels). Heat is supplied to the heat-transfer agent over the channel perimeter; the heat sink is in the central channel.

The problem reduces to finding the nonsteady temperature fields $T(\tau, r, x, \varphi)$ for heat transfer by conduction in the porous structure from the periphery to the central channel. The complexity of the geometry means that the problem cannot be solved analytically. In view of the symmetry it is possible to take one-twelfth of the model of length L (Fig. 1) bounded by adiabatic surfaces. For numerical solution, the calculational model may be divided into cells of sufficiently small cross section to allow the temperature gradient to be ignored within the limits of the cell.

A suggested division of the model into calculational cells is shown in Fig. 1, together with a numbering system for the cell centers, based on an integer (i, j) rectangular coordinate system [1]. In this formulation the problem reduces to the solution of nonsteady one-dimensional (in x) differential heat-conduction equations, the number of which is equal to the number of cells. The calculation heat-transfer equation written for a characteristic cell of triangular packing is

$$C_p\gamma\frac{\partial T_{i,j}(\tau,\ x)}{\partial\tau}=\operatorname{div}(\lambda\nabla T_{i,j}(\tau,\ x))+\frac{1}{F_{i,j}}\left(\sum_{m=1}^{3}q_{h,m}+\sum_{n=1}^{3}q_{k,n}\right). \tag{1}$$

The complete system of equations describing heat transfer to the central channel may be obtained by writing Eq. (1) for all the calculational cells of the packing. It must be complemented by temperature dependences for the thermophysical properties of the constructional material and the heat-transfer agent. The heat fluxes at the boundaries of the calculational cells facing the pore may be written on the basis of the heat-transfer conditions in the form

$$q_{k,n}=\begin{cases}-\lambda_k\Pi_k(\partial T_k/\partial r_k)\,|_{r_k=d/2};\\ \alpha_w\Pi_k(T_k-T_{i,j}(\tau,\ x));\\ \varepsilon\sigma\Pi_k(T_k^4-T_{i,j}(\tau,\ x)).\end{cases} \tag{2}$$

The temperature of the hot source T_k is determined from the nonsteady heat-transfer equation, which for a channel with internal heat sources, for example, takes the form

$$C_{p_k}\gamma_k\frac{\partial T_k}{\partial\tau}=\operatorname{div}(\lambda_k\nabla T_k)+q_v-\sum_{n=1}^{6}\frac{1}{F_k}q_{k,n}. \tag{3}$$

For the case whan a hot heat-transfer agent circulates in the channel the temperature T_k may be determined from the differential equation of convective heat transfer:

$$C_{p_k}\gamma_k\left(\frac{\partial T_k}{\partial\tau}+W\nabla T_k\right)=\operatorname{div}(\lambda_k\nabla T_k)-\sum_{n=1}^{6}\frac{1}{F_k}q_{k,n}. \tag{4}$$

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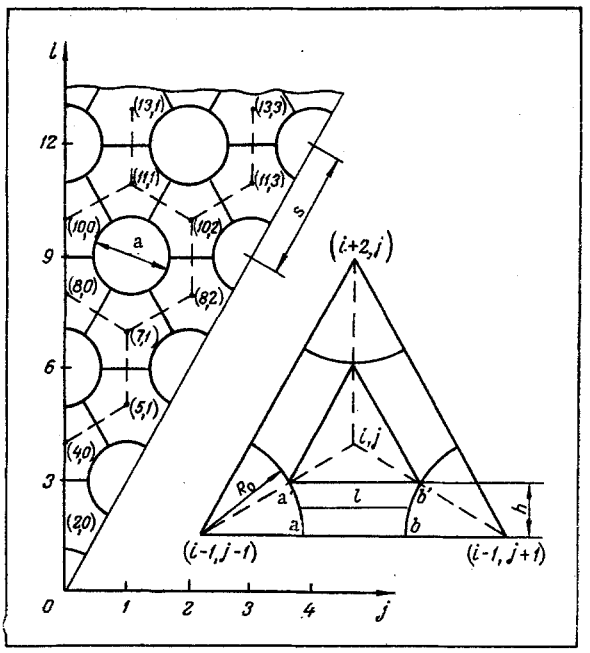

Fig. 1. Calculational model and numbering of cells.

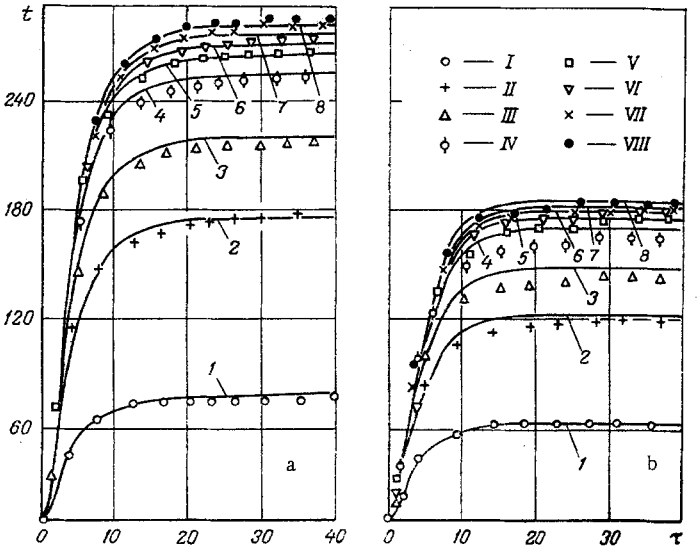

Fig. 2. Nonsteady temperature field in cells of the model and comparison with experimental data: a) s/d = 1.24; b) 1.5; N = 1.8 kW. The continuous curves are calculational relations: 1) for the cell (i = 2, j = 0); 2) (4,0); 3) (5,1); 4) (7,1); 5) (8,0); 6) (8,2); 7) (10,0); 8) (10,2); I) experimental data for the cell (i = 2, j = 0); II) (4,0); III) (5,1); IV) (7,1); V) (8,0); VI) (8,2); VII) (10,0); VIII) (10,2). τ, min; t, °C.

The number of relations of the type in Eqs. (3) and (4) is the same as the number of calculational channels.

In calculating the temperature fields in this system the greatest difficulty attaches to the correct specification of the radial heat-transfer law at the boundaries between adjacent cells. The heat flow for adjacent cells may be written in the form

$$q_{\tau,m} = \frac{k\lambda}{\delta}(T_m - T_{i,j})(s - d), \tag{5}$$

where $k = k_1 k_2$ is a coefficient taking into account the effect of asymmetry of the current lines with respect to the local axis, rotation of the current line by 60° in the limits of the cell, the discrepancy between the well-boundary surface and the effective value, etc.

The coefficient k_1, taking into account the discrepancy of the effective surface from (s − d) L for linear temperature variation over a line containing the centers of adjacent cells, may be calculated from the expression

$$k_1 = \frac{\Pi}{s-d} \approx \frac{1}{s-d} \cdot \frac{\int_0^h l dh}{h} = \frac{\frac{s}{d} - \frac{\pi}{6} - \frac{\sqrt{3}}{4}}{\frac{s}{d} - 1}, \quad (6)$$

where $\int_0^h l dh$ is the area of the curvilinear trapezium $aa'b'b$ (Fig. 1).

The coefficient k_2 takes into account all the other factors and depends on the position of the calculational cell. In the first approximation it may be taken equal to k_1 for all the cells; this approximation has been verified experimentally.

The above system of nonsteady heat-transfer equations was solved numerically on a BÉSM-6 computer using an explicit grid method [2]. The integer coordinate system adopted considerably simplifies the handling of the calculational cells, especially when there are many of them in the model, since it allows an unambiguous transition from the double cell index to a linear index and back.

To verify the basic assumptions of the calculation procedure, experiments were carried out on three models in the form of steel-45 hexahedra of length 150 mm [3]. A hole of diameter 12.8 mm was drilled on the axis of the model to accommodate a cooling-water duct. A triangular lattice of channels was obtained by drilling holes of diameter 9.5, 10.5, and 11.5 mm at a separation of 14.2 mm.

Heat was supplied to the cells of the model by radiation from tubular electric heaters fitted inside the channels. Uniform heat flux over the length of the channels was ensured by maintaining the same power in each heater by connecting their electricity supply in three sections. The outer surface of the model was thoroughly heat-insulated; the maximum heat loss in the course of the experiment was 1.6-7%, depending on the temperature conditions. The temperature was measured in the central cross section of the model using Chromel−Alumel thermocouples with thermoelectrode diameter 0.3 mm, fitted in special holes (diameter 3 mm) drilled at the center of the calculational cells.

The experiments were carried out for a constant power supply to the electrical heaters and constant flow rate of cooling water. Some experimental results and a comparison with calculational data obtained when $k_1 = k_2$ are shown in Fig. 2. The agreement between experiment and calculation is evidently adequate. The maximum discrepancy between the calculated and experimental temperature fields in all conditions did not exceed 5-7% and fell within the limits of experimental error.

Note finally that this method of numerical calculation of nonsteady temperature fields in bodies of complex geometry may successfully be used in the case when the heat-transfer agent in the space between the tubes undergoes a phase transition (for example, heating and melting of the heat carrier). In this case Eqs. (1)-(4) must be augmented by a differential heat-transfer equation describing the limit of displacement of the phase-transition front.

NOTATION

τ, time; x, axial coordinate; r, radial coordinate; φ, angular coordinate; s, packing step; d, channel diameter; Π, heat-transfer perimeter; δ, distance between points of adjacent cells; F, cross-sectional area; W, mean-mass heat-transfer-agent velocity; T, temperature; γ, C_p, λ, density, specific heat, and thermal conductivity; α_w, heat-transfer coefficient; σ, Stefan−Boltzmann coefficient; q_v, bulk heat liberation; $q_{k,n}$, heat influx to cell i, j; $q_{h,m}$, heat flux to cell i, j from adjacent cells; N, thermal power; ε, emissivity; i, j, integer coordinates of calculational cell. Indices: m, cells adjacent to cell i, j; n, channels adjacent to cell i, j; k, channel.

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